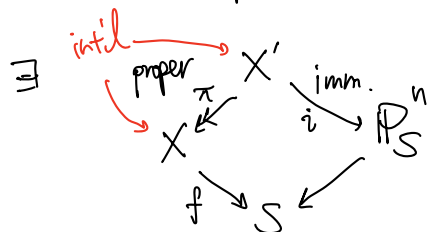


§1. 周炜良引理

Thm (Chow's Lemma). $S = \text{Spec}(A = \text{Noeth. ring})$, $X \xrightarrow{f} S$ sep. + f.type. + X int'l

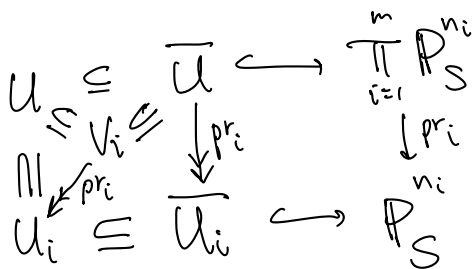
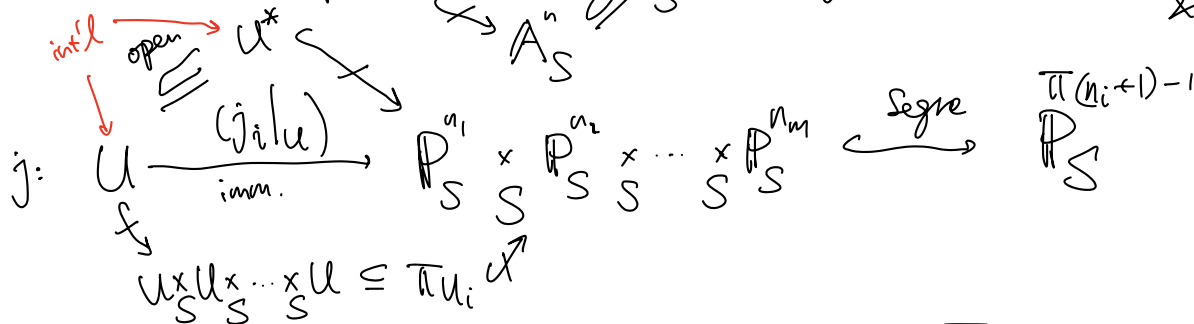


& $\exists U \subseteq X$ dense open s.t. $\pi^{-1}(U) \xrightarrow{\cong} U$.

(f proper $\Rightarrow i$ is a closed imm.)

pf: $X = \bigcup_{i=1}^m U_i$, $U_i := \bigcap_{j=1}^m U_{ij} = \text{Spec}(B_i)$.

For each i , $\text{Spec}(B_i) \xrightarrow{h_i} \mathbb{P}_S^{n_i}$ by lemma before (h_i is f.c. & U_i is reduced).



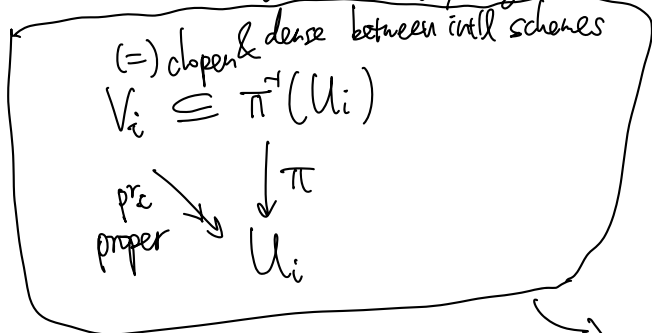
$V_i \subseteq \overline{U}$
 $\hookrightarrow \pi^{-1}(U_i)$.

$U \subseteq X' := \bigcup_{i=1}^m V_i \subseteq U^*$

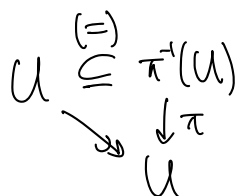
$X' \xrightarrow{\pi} X$: $\pi|_{V_i \cap V_j} = \pi|_{V_i \cap V_j}$ as they agree on $U \subseteq V_i \cap V_j$.

($\text{Eq}(V_i \cap V_j \rightrightarrows X) \cong (V_i \cap V_j) \times_{X \times_S X} X$) is a closed subsch. in $V_i \cap V_j \times_{X \times_S X} X$ which contains U .

$V_i \rightarrow U_i \subseteq X$ glue to get



Similarly



π is proper + surj.

Cor. $f: X \xrightarrow{\text{proper}} Y$, $F \in \text{Goh}(X)$, Y (hence X also) loc. Noeth.

Then $f_{\&}\mathbb{F} \in \text{Coh}(Y)$.

pf: WMA: $Y = \text{Spec}(A)$.

WMA: $Y = \text{Spec}(A)$.
 $\exists$ fil. $0 \subseteq F_0 \subseteq \dots \subseteq F_m = F$, $Z_i \overset{\text{closed}}{\hookrightarrow} X$ int'l, $\mathcal{L}_i = \mathcal{O}_{Z_i}$.

$$\text{st. } \mathbb{F}_i / \mathbb{F}_{i-1} = (Z_i \hookrightarrow X)_* \mathbb{Z}_i.$$

$$0 \rightarrow \Gamma(F_{i-1}) \rightarrow \Gamma(F_i) \rightarrow \Gamma((Z_i \hookrightarrow X)_{\#} \mathcal{I}_i)$$

$$\Gamma(Z_i, \mathcal{I}_i) \hookrightarrow \Gamma(Z_i, \mathcal{O}_{Z_i}) \quad (\hookrightarrow \Gamma(Z'_i, \mathcal{O}))$$

So suffices, by induction, to show $\underbrace{\{ \langle \varepsilon_i, \varepsilon_i \rangle \}}_{\text{fin. A-mod.}}$

Now apply 周同理

$$\begin{array}{ccc} & \pi & \\ & \swarrow & \\ Z_i & & Z'_i \\ & \searrow & \downarrow \text{proj.} \\ & & Y \end{array}, \quad \mathcal{O}_{Z_i} \hookrightarrow \pi_* \mathcal{O}_{Z'_i}.$$

Appeal to $\mathrm{prj}_* \mathcal{O} \in \mathrm{Coh}(Y)$.

§2. Formal function Thm.

Formal function Thm.

$$X \xrightarrow{f} Y \ni y \rightsquigarrow X_{\overset{X}{\uparrow \text{pr}_1}}^x \text{Spec}(\mathcal{O}_{Y,y}) \xrightarrow{\text{pr}_2} \text{Spec}(\mathcal{O}_{Y,y} \hat{=} A) \overset{m}{\uparrow \text{pr}_1}$$

Lemma: $\int \mathbb{Q}^c \mathbb{Q}^S, F \in \mathcal{QCoh}(X), (f_*^L F)_y \cong \Gamma(X_y^x \times_{\text{Spec}(A)}, pr_1^* F).$

2 completions : $\left((f, F)_y \right)^{\wedge}_m := \lim_n \Gamma(X_{\tilde{Y}}^x \text{Spec}(A), \text{pr}_1^* F) / m^n$
 always arrow $\downarrow \text{Comp}(f, F, y)$
 $\lim_n \Gamma(X_{\tilde{Y}}^x \text{Spec}(A/m^n), \text{pr}_1^* F)$

Thm* (FFT): If f proper, γ (loc.) Noeth., F coh., then.

$\text{Comp}(f, F, y)$ is an isom.

§3. Stein factorization.

Thm: $S = \text{loc. Noeth.}$, $X \xrightarrow{f} S$ proper, set

$$\begin{array}{ccc} X & \xrightarrow{f'} & \text{Spec } \mathbb{A}_k \\ & \searrow f & \swarrow \pi \\ & S & \end{array} \quad f_* \mathcal{O}_X = S' \quad \text{Then}$$

- (1) π finite

$$(2) \quad f'_* \mathcal{O}_X = \mathcal{O}_{S'}$$

(3) f' is proper

(4) f' has conn'd fibers. (note that ϕ is not conn'd.)

pf: (1) - (3) exercise.

For (4): $f_* \mathcal{O}_X = \mathcal{O}_S \Rightarrow$ conn'd fiber.

Suppose $X_S = Y \sqcup Z$, then so is $X \times_S \text{Spec}(\mathcal{O}_{S,s}/\mathfrak{m}_s^n)$.
 " $X \times_S \text{Spec}(K(S)) \cong Y_n \sqcup Z_n$.
 (b/c top. space agrees).

$$\Rightarrow (f_{*}\mathcal{O}_X)^{\wedge}_s = \mathcal{O}_{S,s}^{\wedge}$$

$$\stackrel{\text{FFT}}{=} \lim_n \Gamma(Y_n \perp Z_n, \emptyset) = \text{nonzero rfp} \times \text{nonzero rfp}.$$

~~_____~~

Cor. (Baby version of ZMT).

$X \xrightarrow{f} Y$, proper between loc. Noeth. integral schemes. Assume:

(bivert.) (1) $f|_{\eta_X}: \eta_X \xrightarrow{\cong} \eta_Y$ $(\iff (1'): \exists u \in Y \text{ s.t. } f^{-1}(u) \xrightarrow{\cong} u)$.

(2) Y is normal. ($\mathcal{O}_{Y,y}$ are int'ly closed domains $\forall y \in Y$).

Then f has conn'd fibers.

pf: $\mathcal{O}_Y \rightarrow f_* \mathcal{O}_X$ finite + inj. + same function field,
hence = by normality of Y .